An impossibility theorem of aggregating semantic rankings under non-monotonic quantification

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Outline

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Scalar items

Scalar items have **a logically weaker** and **a logically stronger** meaning.

Examples of lexical scales:

1. ⟨ some, all ⟩

   "Some" is a scalar item with:

   - a weaker meaning: **at least one** (= literal meaning)
   - a stronger meaning: **some but not all** (= strengthened meaning, through **competition with "all"**)

2. ⟨ a book, books ⟩

   "Books" is a scalar item with:

   - a weaker meaning: **at least one** book (= literal meaning)
   - a stronger meaning: **at least two** books (= strengthened meaning, through **competition with "a book"**)

Scalar items under quantifiers

Sentences with scalar items **in the scope of quantifiers** have non-trivial truth conditions.

Example:

(1) **Each** box contains books.

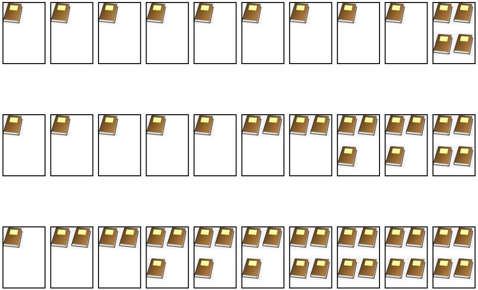
is true...

- iff each box has **at least one** book?
- iff each box has **at least two** books?
- under some other conditions...?

Motivation

Empirical data from previous work (Rong and Spector 2026) using sentences with **scalar items** under **universal quantifiers:**

(2) **Each** box contains books.



Motivation

Best statistical model predicting the graded acceptability judgments of the sentence against different distributions:

$$\text{response} \sim \underbrace{c_{\text{lit}} + c_{\text{str}}}_{\substack{\text{binary predictors} \\ \text{(whether certain readings} \\ \text{are satisfied)}}} + \underbrace{c_{\text{vrf}}}_{\text{gradient predictor}} + (1 \mid \text{participant})$$

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Two types of factors contribute to felicity judgments:

categorical = reading is true/false

gradience = proximity to the 'best' scenario

Speakers **classify** situations but also **rank** them.

Non-monotonic quantifier *exactly n*

(3) **Exactly two** boxes contains books.

Two complications:

1. situations can satisfy the locally strengthened interpretation (*exactly two boxes contains several*) without satisfying the literal one (*exactly two…at least one*)
2. proximity to 'best' case is also proximity to false cases

How should felicity judgments be organized in the non-monotonic case?

Two conjectures for ranking situations

1. Based on **trivalence + distance**
   - true / false / undefined
   - undefined cases ordered by distance to best case
2. Based on **satisfaction of readings**
   - three bivalent readings: literal / intermediate / local
   - more satisfied readings = better

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A meta-theoretic question

Can a speaker have a principled way of forming a single felicity judgment that reconciles several rankings?

Outline

Situations

(4) **Exactly** n [individuals] [P].

where P is a predicate containing a scalar item.

Each individual stands in a certain relation to P:

F $=$ falsifier

W $=$ weak verifier (verifies literal but not strengthened meaning of P)

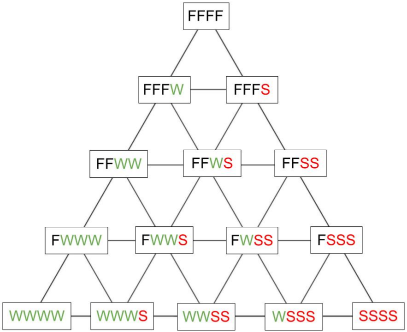
S $=$ strong verifier (verifies strengthened meaning of P)

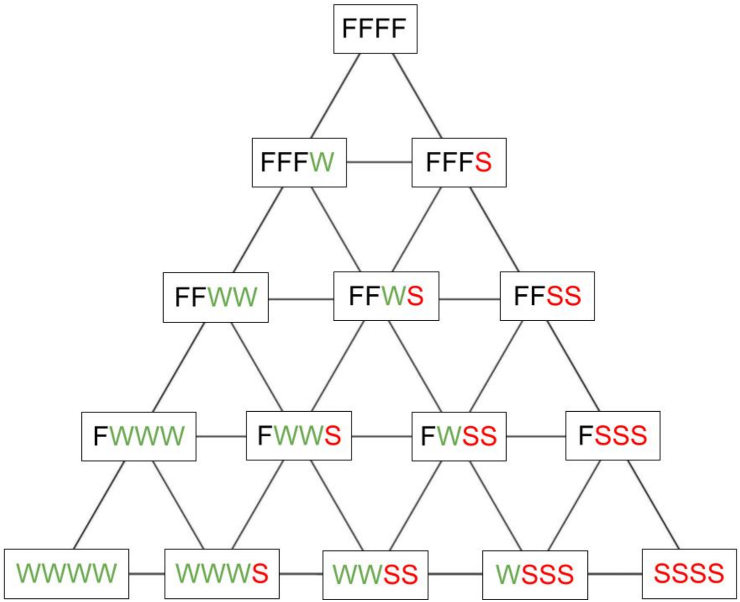
$n = 2$, with domain of 4 individuals

(5) **Exactly two** [boxes] [contain books].

F is zero books
W is ≥ 1 book
S is ≥ 2 books

FFSS = intuitively 'best' case
FWSS = slightly worse
WWSS = worse

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Order-theoretic perspective

Every theory induces a **ranking**.

We will model a ranking as **total preorder**,
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If φ, χ and ψ are situations, we have:

Reflexivity	$\varphi \succeq \varphi$
Transitivity	If $\varphi \succeq \chi$ and $\chi \succeq \psi$, then $\varphi \succeq \psi$.
Connectedness	$\varphi \succeq \chi$ or $\chi \succeq \varphi$

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Strict relation $\succ$ defined by: $\varphi \succ \chi$ iff $\varphi \succeq \chi$ and not $\chi \succeq \varphi$.

Induced equivalence relation $\approx$ defined by: $\varphi \approx \chi$ iff $\varphi \succeq \chi$ and $\chi \succeq \varphi$.

First ranking: trivalence + distance

(6) **Exactly** *n* [individuals] [*P*].

Trivalent truth-values from Strong Kleene:

- $\mathbb{T}$: exactly *n* individuals are S, all others are **F**.
- $\mathbb{F}$: either less than *n* individuals in total are S or W, or than more than *n* individuals in total are S or W and among them not exactly *n* individuals are S.
- $\mathbb{U}$: otherwise.

Trivalent distance-based ranking

Define a weak relation $\succeq_t$ on situations by $\varphi \succeq_t \chi$ iff:
(i) $V(\varphi) \geq V(\chi)$, where the order on truth-values is $\mathbb{T} > \mathbb{U} > \mathbb{F}$,
(ii) when both φ and χ are $\mathbb{U}$, φ is closer than χ to the closest $\mathbb{T}$-situation.

Idea: gradient proximity to optimal truth.

Second ranking: satisfaction of readings

Three bivalent readings:

Literal: exactly n are at least weak verifiers.

Intermediate: exactly n individuals are at least weak verifiers and **it is not the case** that all of the n individuals are weak verifiers.

Local: exactly n are strong verifiers.

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Reading-satisfaction-based ranking

Define the weak reading-satisfaction relation $\succeq_r$ by $\varphi \succeq_r \chi$ iff:

either $\#(\varphi) > \#(\chi)$

or $\#(\varphi) = \#(\chi)$, and if $\#(\varphi) = 1$, then φ verifies the literal reading whenever χ verifies the local reading[1]

[1] Empirical data shows salience of the literal reading over the local reading.

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Idea: more readings satisfied = better.

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We now have **two rankings over the same set of situations.**

An aggregation problem adapted for semantics

Can a speaker entertain both rankings but still produce a single judgment, while satisfying a small set of natural consistency constraints?

The aggregation problem

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An aggregation problem adapted for semantics

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Original aggregation problem in social choice

How can we combine many individual preference rankings into a single coherent collective ranking?

Analogy: Social choice theory

Functions that aggregate different rankings into a unified 'social' ranking have been studied thoroughly in the theory of **social choice**...

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(1743-1794)

Lewis Carroll
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and applied beyond preference aggregation:

- Confirmation theory: Stegenga 2013
- Verisimilitude: Zwart and Franssen 2007; Morreau 2010
- Biodiversity: Süskind 2026

Condorcet's paradox

Three voters A, B, C, are asked to rank three choices, φ, χ, ψ.

Their rankings are:

	A	B	C
1	φ	ψ	χ
2	χ	φ	ψ
3	ψ	χ	φ

Write $\varphi \succeq \psi$ if a majority of voters prefers φ over ψ.
Then:

- $\chi \succeq \varphi$, and $\psi \succeq \chi$ (and by transitivity, $\psi \succeq \varphi$)
- but also $\varphi \succeq \psi$...

Arrow's axioms and theorem

Aggregation function:
takes several rankings as input and returns one ranking as output.

An aggregation function that outputs a ranking $\succeq$ should satisfy the following conditions:

Unanimity: If every voter prefers φ over ψ, then $\varphi \succeq \psi$.

Independence: Adding a new option χ cannot overturn previous relative rankings.

Non-dictatorship: No voter D is such that $\varphi \succeq \psi$ whenever D prefers φ over ψ.

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Impossibility theorem (Arrow 1951)

If at least 2 voters rank at least 3 options, then there exists no aggregation function which simultaneously satisfies Unanimity, Independence, and Non-dictatorship.

Outline

Back to *exactly n*

$$\begin{aligned} \text{Candidates} &\quad\simeq\quad \text{situations to be ranked} \\ \text{Voters} &\quad\simeq\quad \text{semantic theories} \\ \text{Aggregation function} &\quad\simeq\quad \text{`the mind'} \end{aligned}$$

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2. Arrow's *Independence* compares the same pair of candidates across different voting profiles. **But our setting has a single pair of semantic rankings.**
   We replace *Independence* by:
   Locality: Any two pairs of situations exhibiting the same comparison pattern under $\succeq_r$ and $\succeq_t$ must receive the same comparison under the unified ranking.

Strategy of the proof

- By Locality: everything reduces to **pairs of situations**.
- Consider a pair φ, χ. There are only 3 relevant cases:
  - disagreement between $\succeq_t$ and $\succeq_r$
  - $\succeq_t$ neutral, $\succeq_r$ strict
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$\Rightarrow$ **Finitely many possible aggregation rules.** *sigh of relief*

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The only 3 configurations that matter:

A. $\langle \varphi \succ_r \chi,\ \chi \succ_t \varphi \rangle$ **B.** $\langle \varphi \succ_r \chi,\ \varphi \approx_t \chi \rangle$ **C.** $\langle \varphi \approx_r \chi,\ \varphi \succ_t \chi \rangle$

For each of these profiles, the unifying ranking must select one of 3 outcomes:

1. $\varphi \succ \chi$ **2.** $\chi \succ \varphi$ **3.** $\varphi \approx \chi$

$\Rightarrow 3^3 = 27$ formally possible ways of defining a unifying ranking.
We eliminate them one by one.

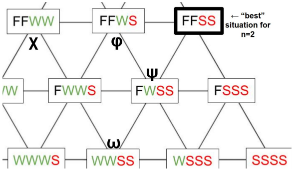
One elimination example

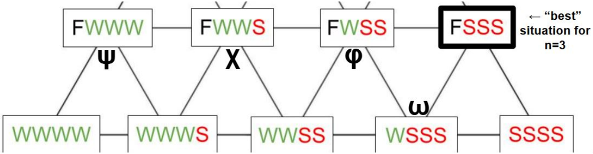
Lemma

One of these two profiles is always instantiated:

$$\langle \varphi \succ_r \chi \succ_r \psi \approx_r \omega,\ \varphi \approx_t \psi \succ_t \chi \approx_t \omega \rangle \qquad (P_1)$$

$$\langle \varphi \approx_r \chi \succ_r \psi \succ_r \omega,\ \varphi \approx_t \omega \succ_t \chi \succ_t \psi \rangle \qquad (P_2)$$





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Suppose profile **A** is mapped to $\varphi \succ \chi$ and **B** to $\chi \succ \varphi$.
Consider one of the two profiles described in the Lemma:

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B gives $\psi \succ \varphi$. By transitivity: $\boldsymbol{\chi \succ \varphi}$.

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B gives $\psi \succ \varphi$. By transitivity: $\boldsymbol{\chi \succ \varphi}$.
But Unanimity gives $\varphi \succ \chi$!

$$\langle \varphi \succ_r \chi \succ_r \psi \approx_r \omega, \varphi \approx_t \psi \succ_t \chi \approx_t \omega \rangle$$

Impossibility result and interpretation

Lemma (repeated)

One of these two profiles is always instantiated:

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Interim result after eliminations:

1. Profile (P_1) excludes everything except A3-B1-C1.
2. Profile (P_2) excludes everything except A2-B2-C2 and A2-B2-C3.

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Finishing the proof:
In Python, computationally verify if the remaining candidates induce a total preorder on full situation spaces (domain size up to 10).
$\rightarrow$ Output: the induced relation is **not transitive.**

Impossibility result

Theorem

There exists no unifying function that aggregates, for all *exactly n* with $n \geq 2$, the reading-satisfaction-based ranking and the trivalent distance-based ranking into a total preorder while satisfying:

- Unanimity
- Locality
- Non-dictatorship

Impossibility result

Theorem

There exists no unifying function that aggregates, for all *exactly n* with $n \geq 2$, the reading-satisfaction-based ranking and the trivalent distance-based ranking into a total preorder while satisfying:

- Unanimity
- Locality $\leftarrow$ least cognitive plausible among the three
- Non-dictatorship

Where to go from here?

One option: relax Locality.

Take-home message

- Quantified sentences give rise to competing ways of ranking situations.
- Empirical evidence suggests that speakers' felicity judgments may integrate aspects of two theories.
- If speakers integrate two rankings into a single felicity judgment, this becomes an aggregation problem.
- An Arrow-style impossibility theorem: under three consistency principles, no aggregation is possible.

Take-home message

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- Empirical evidence suggests that speakers' felicity judgments may integrate aspects of two theories.
- If speakers integrate two rankings into a single felicity judgment, this becomes an aggregation problem.
- An Arrow-style impossibility theorem: under three consistency principles, no aggregation is possible.

- Future directions:
  - Relax one of the axioms (especially Locality).
  - Introduce weighted edges, costs, or perturbations.

Special thanks to:
Prof. Benjamin Spector, Prof. Amir Anvari, Jakob Süskind, Léo Wang.

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Thank you for listening!

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