Less is more: a non-verbal approach to anti-exhaustivity

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Claire Rong   Shiyun (Iris) Dong
clairerongfr@gmail.com   shiyun.dong@ens.psl.eu

Based on joint work with Prof. Benjamin Spector

Department of Cognitive Studies, École Normale Supérieure, Paris

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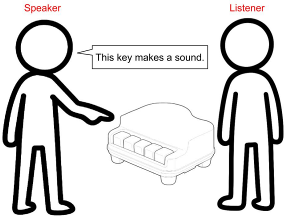
Outline

Introduction: Exhaustivity in verbal communication



Exhaustivity: "This key makes a sound" $\rightsquigarrow$ Only this key makes a sound.

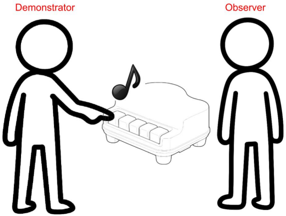
Two approaches to strengthening:

1. Gricean: speaker's intent
2. Grammatical: silent "only" operator (Chierchia, Fox, and Spector 2012)

Experimental results: Cremers, Wilcox, and Spector 2023

1. Use models from the Rational Speech Act framework (Frank and Goodman 2012). Show that many RSA models predict not only exhaustivity, but also anti-exhaustivity when priors are skewed.
   Anti-exhaustivity in a nutshell: for a speaker, it means using a less informative message to trigger a more informative inference; for a listener, it means to assign a posterior probability to the less informative message higher than the prior probability.
2. Present language production and comprehension experiment data
   $\rightarrow$ no trace of anti-exhaustivity.
3. Results indicate that subjects are not rational in a Bayesian sense.

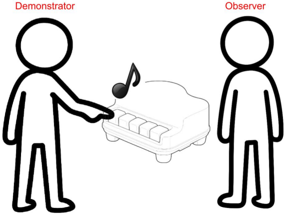
Introduction: Anti-exhaustivity in non-verbal communication



Instead of speaking, the speaker demonstrates with an action. They press only one key, and it makes a sound.

The fact that the key makes a sound $\not\rightsquigarrow$ Only that key makes a sound.

There is **no exhaustivity inference**. But anti-exhaustivity **is not just the absence of exhaustivity!** The observer might think that the demonstrator purposefully pressed only one key because all keys have the same behavior.



Anti-exhaustivity conditions for the observer

P(all keys make a sound | the demonstrator's action) ← posterior probability

> P(all keys make a sound | 'This key makes a sound" is true) ← prior probability

Anti-exhaustivity **on the demonstrator's part:** knows that all keys make a sound, but does not press all of them after considering **tradeoff between informativity and cost**.

Intuitive predictions

1. Anti-exhaustivity arises more easily in non-verbal than in verbal communication.
2. If the observer has a high prior that all keys make a sound, observing that one key makes a sound will confirm their prior.
   Likewise, if the observer has a low prior that all keys make a sound.

Do these intuitions align with the predictions of the baseline RSA model?

Outline

Scenario setup

- Toy with two identical keys K_1 and K_2.
- Simplifying assumption: "K_1 makes a sound" is tautological.
- Two possible **worlds**:
  - $w_{\{1\}}$: only K_1 makes sound.
  - $w_{\{1,2\}}$: both keys make sound.
- Two possible **actions** by the demonstrator:
  - a_1: press only K_1.
  - $a_{\{1,2\}}$: press both keys.

Intuitive predictions

1. In $w_{\{1,2\}}$, if the demonstrator thinks the observer has a high prior $P(w_{\{1,2\}})$, they will only press one key to convey that both keys make sound.
2. If the demonstrator only presses one key, the observer will infer that both keys make a sound if their prior $P(w_{\{1,2\}})$ was above $\frac{1}{2}$, and will **not** infer that the second key makes a sound if $P(w_{\{1,2\}}) < \frac{1}{2}$.

Framework: literal listener L_0

Literal listener L_0

For an utterance u and a world w:

$$L_0(w|u) = P(w|[\![u]\!]) = \begin{cases} \dfrac{P(w)}{P([\![u]\!])} & \text{if } w \in [\![u]\!] \\ 0 & \text{else} \end{cases}$$

where:

- $[\![u]\!]$ = set of worlds where u is true
- $P(w)$ = prior probability of world w

Framework: pragmatic speaker S_1 and pragmatic listener L_1

Utility function

$$U_1(u, w) = \log(L_0(w|u)) - c(u)$$ where $c(u)$ is the cost of utterance.

Pragmatic speaker S_1

$$S_1(u|w) = \frac{\exp\left(\lambda U_1(u, w)\right)}{\sum_{u'} \exp\left(\lambda U_1(u', w)\right)}$$ where λ is a rationality parameter.

The softmax function models the speaker as **approximately rational**: more useful utterances are more likely to be chosen.

Framework: pragmatic speaker S_1 and pragmatic listener L_1

Utility function

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$$S_1(u|w) = \frac{\exp\left(\lambda U_1(u, w)\right)}{\sum_{u'} \exp\left(\lambda U_1(u', w)\right)} \quad \text{where } \lambda \text{ is a rationality parameter.}$$

The softmax function models the speaker as **approximately rational**: more useful utterances are more likely to be chosen.

Pragmatic listener L_1

$$L_1(w|u) = \frac{P(w) S_1(u|w)}{\sum_{w'} P(w') S_1(u|w')}$$

Equivalent to **Bayes' rule**: the listener knows the speaker's strategy and combines their **prior belief** $P(w)$ about possible meanings with the **likelihood** $S_1(u|w)$.

Anti-exhaustivity conditions (speaker)

Speaker condition

$$S_1(a_1|w_{\{1,2\}}) > S_1(a_{\{1,2\}}|w_{\{1,2\}}) \quad \text{iff} \quad \underbrace{-\log(P(w_{\{1,2\}}))}_{\text{informativity of pressing } K_2} < \underbrace{c\left(a_{\{1,2\}}, w_{\{1,2\}}\right)}_{\text{cost of pressing } K_2}$$

- **Investing additional cost to press one more key is not justified by the gain in informativity.**
- Speaker prefers under-informative action when:
  - High prior $P(w_{\{1,2\}})$ (keys likely both make sounds)
  - Cost of full demonstration $c(a_{\{1,2\}}, w_{\{1,2\}})$ is high
- Non-verbal case:
  - $P(w_{\{1,2\}}) \approx 1$ (identical keys)
  - $-\log(P(w_{\{1,2\}})) \approx 0 \rightarrow$ condition typically satisfied

Anti-exhaustivity conditions (listener)

Listener condition

$$L_1(w_{\{1,2\}}|a_1) > P(w_{\{1,2\}})$$

iff

$$\underbrace{-\log(P(w_{\{1,2\}})) - (-\log(P(w_{\{1\}})))}_{\text{difference in informativity}} < \underbrace{c(a_{\{1,2\}}, w_{\{1,2\}}) - c(a_{\{1,2\}}, w_{\{1\}})}_{\text{difference in cost}}$$

- **Breaking the symmetry between the two maximally informative actions: if we are in a counter-intuitive world, the speaker would incur greater loss by not being maximally informative.**
- Non-verbal case simplifies to $P(w_{\{1,2\}}) > P(w_{\{1\}})$, which typically holds for identical keys...

  $\rightarrow$ slightly problematic prediction for greater number of keys.

Comparison between speaking and showing

Non-verbal

- Actions cannot be negated.
- Cost depends on action and on world.
- Cost difference $= 0$
- Anti-exhaustivity when:

$$P(w_{\{1,2\}}) > P(w_{\{1\}})$$

$\rightarrow$ easily satisfied

Verbal

- Messages can be negated.
- Cost depends only on utterance.
- Cost difference > 0:
  - "These keys make sound" (low cost)
  - "Only this key makes sound" (higher cost)

$\rightarrow$ harder to satisfy the same inequality

Takeaway

Non-verbal demonstrations naturally lead to anti-exhaustive inferences under identical keys assumption.

Outline

Scenario setup

We extend the previous scenario to n identical keys $K_1, \ldots, K_n$.

- Simplified notations:
  - w_{all}: world where all keys make a sound.
  - $w_{\text{all but one}}$: world where all keys except K_n make a sound.
- Possible actions:
  - $a_{\text{all but one}}$: press first $n-1$ keys.
  - a_{all}: press all n keys.

Intuitive predictions for "all keys but one", with n keys

Does anti-exhaustivity still arise for the demonstrator or the observer when all keys but one are pressed, but the total number of keys is larger?

Demonstrator behavior

As n increases:

- Pressing one more key after $n-1$ becomes relatively cheaper.
- Anti-exhaustivity becomes less likely.

Observer interpretation

Seeing $n-1$ keys pressed:

- Why skip the last one after pressing so many?
- Anti-exhaustivity also less likely.

$\rightarrow$ **How does the cost function shape reflect these behaviors?**

Cost function typology

Cost function (adapted)

$c(a_I, w_J) = c_0 + f(|I|)$ where:

- a_I is the action of pressing exactly the keys $\{K_i\}_{i \in I}$
- w_J where $J \subseteq \{1, \ldots, n\}$ is the set of keys that produce sounds

- **Cost-averse demonstrator**:
  - f convex (e.g., quadratic)
  - Marginal cost increases with more keys
  - "Increasingly lazy" behavior
- **Cost-indifferent demonstrator**:
  - f concave (e.g., radical)
  - Marginal cost decreases with more keys
  - "Thorough" behavior

Generalized conditions for anti-exhaustivity

Speaker condition

$$S_1(a_{\text{all but one}} | w_{\text{all}}) > S_1(a_{\text{all}} | w_{\text{all}})$$

iff

$$\underbrace{-\log(P(w_{\text{all}}))}_{\text{informativity of pressing } K_n} < \underbrace{c(a_{\text{all}}, w_{\text{all}}) - c(a_{\text{all but one}}, w_{\text{all}})}_{\text{cost of pressing } K_n}$$

- **Cost-averse** (f convex):
  - $f(n) - f(n-1)$ increases with n
  - Inequality easier to satisfy
- **Cost-indifferent** (f concave):
  - $f(n) - f(n-1)$ decreases with n
  - Inequality harder to satisfy

Generalized conditions for anti-exhaustivity

Listener condition

$$L_1(w_{\text{all}} | a_{\text{all but one}}) > P(w_{\text{all}})$$

iff

$$\underbrace{-\log(P(w_{\text{all}})) - (-\log(P(w_{\text{all but one}})))}_{\text{difference in informativity}} < \underbrace{c(a_{\text{all}}, w_{\text{all}}) - c(a_{\text{all}}, w_{\text{all but one}})}_{\text{difference in cost}}$$

- In the baseline model:
  - Always holds for identical keys
  - Predicts constant anti-exhaustivity
- **But contradicts intuition when n is large...**

Outline

Overview

Baseline RSA model implemented in Python.

Parameters:

- Total keys: $n = 100$
- Keys pressed: $n - 1$
- Rationality parameter: $\lambda = 3$ (from original paper Cremers, Wilcox, and Spector 2023)

Cost function implementation

Cost-averse (Convex)

$$f(k) = ak^2 + bk + c$$

Cost-Indifferent (Concave)

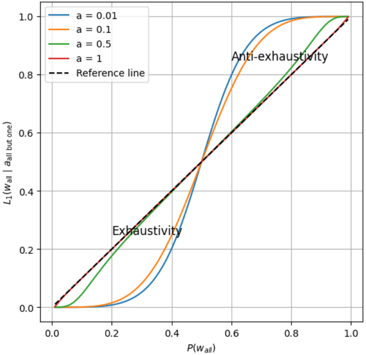
$$f(k) = a\sqrt{k} + b$$

Normalized cost

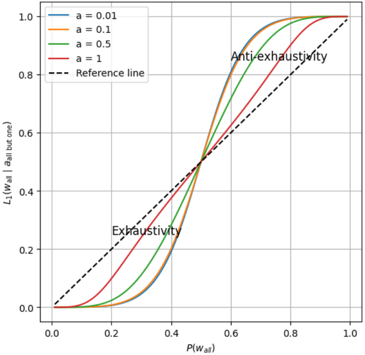
$$F(k) = \frac{f(k)}{f(n)}$$

$F(n) = 1$ for all cases
$\rightarrow$ enables comparison between cost-averse and cost-indifferent.

Simulation for “all keys but one”



(a) Cost-averse



(b) Cost-indifferent

When the observer is biased towards w_{all}, they are more prone to interpret the demonstrator’s actions anti-exhaustively; conversely, they all less prone towards anti-exhaustivity if biased towards $w_{\text{all but one}}$.
Mathematically, this means the inflection point is always 0.5.

Outline

Conclusion and perspectives

Intuitive predictions

- ✓ Anti-exhaustivity arises more easily in non-verbal than in verbal communication.
- ✓ If the observer has a high prior that all keys make a sound, observing that one key makes a sound will not change their prior.
  Likewise, if the observer has a low prior that all keys make a sound.
  $\rightarrow$ But intuitions are not captured by the model for $n-1$ keys with great values of n. Intuitively, anti-exhaustivity should arise **much less easily**, both for the demonstrator and the observer.

Perspectives:

- Refine the model to incorporate the ‘penalty’ of pressing keys ‘for nothing’ and to better capture the intuition for $n-1$ keys with great values of n.
- Explore more complex RSA models.
- Alternative set-ups with objects that do not have an expected behavior given our world knowledge.
- Empirical validation.

Thank you!

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References I

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